Advances in Probabilistic Model Checking

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Probabilistic models

	Fully probabilistic	Nondeterministic
Discrete time	Discrete-time Markov chains (DTMCs)	Markov decision processes (MDPs) (probabilistic automata)
Continuous time	Continuous-time Markov chains (CTMCs)	Probabilistic timed automata (PTAs)
		CTMDPs/IMCs

Overview

- Lecture 3
  - Introduction
  - 1 – Discrete time Markov chains
  - 2 – Markov decision processes
  - 3 – Compositional probabilistic verification
  - 4 – Probabilistic timed automata
- Course materials available here:
  - http://www.prismmodelchecker.org/courses/marktoberdorf11/
  - lecture slides, reference list, exercises

Part 3

Compositional probabilistic verification

Overview (Part 3)

- Compositional verification
  - assume-guarantee reasoning
- Markov decision processes
  - probabilistic safety properties
  - multi-objective model checking
- Probabilistic assume guarantee
  - semantics, model checking
  - assume-guarantee proof rules
  - quantitative approaches
  - implementation & experimental results
  - assumption generation with learning

Compositional verification

- Goal: scalability through modular verification
  - e.g. decide if $M_1 \,||\, M_2 \models G$
  - by analysing M_1 and M_2 separately
- Assume-guarantee (AG) reasoning
  - use assumptions A about the context of a component M
  - $\langle A \rangle\ M\ \langle G \rangle$ – "whenever M is part of a system that satisfies A, then the system must also guarantee G"
  - example of asymmetric (non-circular) AG rule:

$$\frac{\langle \mathrm{true} \rangle\ M_1\ \langle A \rangle \qquad \langle A \rangle\ M_2\ \langle G \rangle}{\langle \mathrm{true} \rangle\ M_1 \,||\, M_2\ \langle G \rangle}$$

[Pasareanu/Giannakopoulou/et al.]

AG rules for probabilistic systems

- How to formulate AG rules for Markov decision processes?

$$\frac{\langle true \rangle\ M_1\ \langle A \rangle \qquad \langle A \rangle\ M_2\ \langle G \rangle}{\langle true \rangle\ M_1 \,||\, M_2\ \langle G \rangle}$$

- Questions:
  - What form do assumptions and guarantees take?
  - What does $\langle A \rangle\ M\ \langle G \rangle$ mean? How to check it?
  - Any restriction on parallel composition $M_1 \,||\, M_2$?
  - Can we do this in a "quantitative" way?
  - How do we generate suitable assumptions?

AG rules for probabilistic systems

- How to formulate AG rules for Markov decision processes?

$$\frac{\langle \text{true} \rangle\ M_1\ \langle A \rangle \qquad \langle A \rangle\ M_2\ \langle G \rangle}{\langle \text{true} \rangle\ M_1 \,||\, M_2\ \langle G \rangle}$$

- Questions:
  - What form do assumptions and guarantees take?
    - probabilistic safety properties
  - What does $\langle A \rangle\ M\ \langle G \rangle$ mean? How to check it?
    - reduction to multi-objective probabilistic model checking
  - Any restriction on parallel composition $M_1 \,||\, M_2$?
    - no: arbitrary parallel composition
  - Can we do this in a "quantitative" way?
    - yes: generate lower/upper bounds on probabilities
  - How do we generate suitable assumptions?
    - learning techniques (L* algorithm)

Overview (Part 3)

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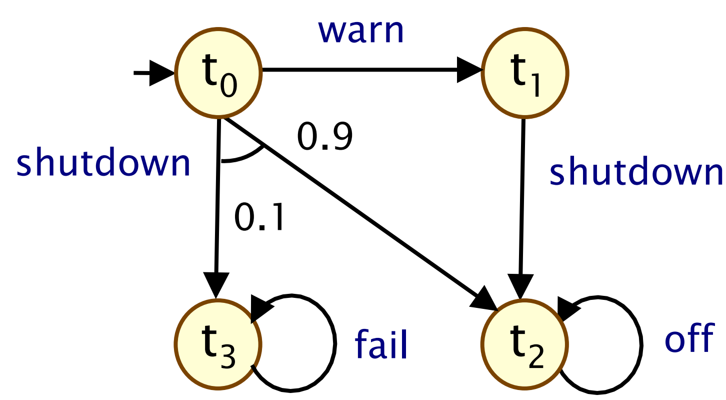
  - quantitative approaches
  - implementation & experimental results
  - assumption generation with learning

Recap: Markov decision processes

- Markov decision processes (MDPs)
  - model probabilistic and nondeterministic behaviour
- An MDP is a tuple $M = (S, s_{init}, \alpha_M, \delta_M, L)$:
  - S is the state space
  - $s_{init} \in S$ is the initial state
  - α_M is the action alphabet
  - $\delta_M \subseteq S \times (\alpha_M \cup \tau) \times Dist(S)$ is the transition probability relation
  - $L : S \rightarrow 2^{AP}$ labels states with atomic propositions



- Notes:
  - α_M, δ_M have subscripts to avoid confusion with other automata
  - transitions can also be labelled with a "silent" τ action
  - we write $s \xrightarrow{a} \mu$ as shorthand for $(s,a,\mu) \in \delta_M$
  - MDPs, here, are identical to probabilistic automata [Segala]

Recap: Adversaries for MDPs

- Adversaries resolves the nondeterminism in MDPs
  - also called “schedulers”, “strategies”, “policies”, …
  - make a (possibly randomised) choice, based on history
- An adversary σ for an MDP M
  - induces probability measure $Pr_{M,s}^{\sigma}$ over (infinite) paths $Path_{M,s}^{\sigma}$
  - we will abbreviate $Pr_{M,s_{init}}^{\sigma}$ to Pr_{M}^{σ} (and $Path_{M,s_{init}}^{\sigma}$ to $Path_{M}^{\sigma}$)
- For adversary σ, we can compute the probability…
  - … of some measurable property ϕ of paths
  - here, we use either temporal logic (LTL) over state labels
    - e.g. $\Diamond err$ – “an error eventually occurs”
    - e.g. $\Box(req \rightarrow \Diamond ack)$ – “req is always followed by ack”
  - or automata over action labels (see later)
    - e.g. deterministic finite automata (DFAs)

Recap: Model checking for MDPs

- Property specifications: quantify over all adversaries
  - e.g. $M \models P_{\geq p}[\phi] \;\Leftrightarrow\; Pr_M^{\sigma}(\phi) \geq p$ for all adversaries $\sigma \in Adv_M$
  - corresponds to best-/worst-case behaviour analysis
  - requires computation of $Pr_M^{min}(\phi) = \inf_{\sigma} \{ Pr_{M,s}^{\sigma}(\phi) \}$ or $Pr_M^{max}(\phi) = \sup_{\sigma} \{ Pr_{M,s}^{\sigma}(\phi) \}$
  - or in a more quantitative fashion:
  - just ask e.g. $P_{min=?}(\phi)$ or $P_{max=?}(\phi)$

- Model checking: efficient algorithms exist
  - for reachability, graph-based analysis + linear programming
  - in practice, for scalability, often approximate (value iteration)
  - for LTL, first do reachability an automaton-MDP product
  - implemented in tools like PRISM, Liquor, RAPTURE

Parallel composition for MDPs

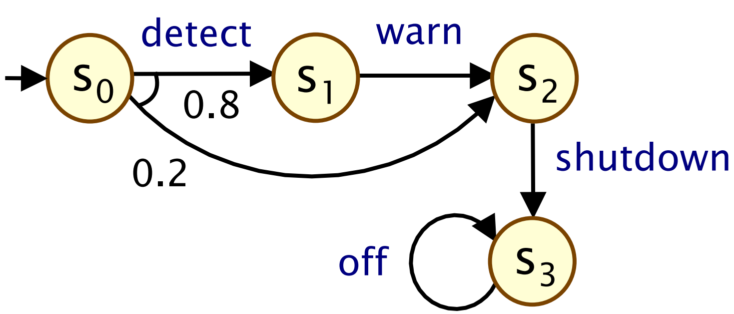
- The parallel composition of M_1 and M_2 is denoted $M_1 \mid\mid M_2$
  - CSP style: synchronise over all common (non-τ) actions
  - when synchronising, transition probabilities are multiplied

- Formally, if $M_i = (S_i, s_{init,i}, \alpha_{M_i}, \delta_{M_i}, L_i)$ for $i=1,2$, then:
- $M_1||M_2 = (S_1 \times S_2, (s_{init,1}, s_{init,2}), \alpha_{M_1} \cup \alpha_{M_2}, \delta_{M_1||M_2}, L_{12})$ where:
  - $L_{12}(s_1,s_2) = L_1(s_1) \cup L_2(s_2)$
  - $\delta_{M_1||M_2}$ is defined such that $(s_1,s_2) \xrightarrow{a} \mu_1 \times \mu_2$ iff one of:
    - $s_1 \xrightarrow{a} \mu_1$, $s_2 \xrightarrow{a} \mu_2$ and $a \in \alpha_{M_1} \cap \alpha_{M_2}$ (synchronous)
    - $s_1 \xrightarrow{a} \mu_1$, $\mu_2 = \eta_{s_2}$ and $a \in (\alpha_{M_1} \backslash \alpha_{M_2}) \cup \{\tau\}$ (asynchronous)
    - $s_2 \xrightarrow{a} \mu_2$, $\mu_1 = \eta_{s_1}$ and $a \in (\alpha_{M_2} \backslash \alpha_{M_1}) \cup \{\tau\}$ (asynchronous)
  - where $\mu_1 \times \mu_2$ denotes the product of distributions μ_1, μ_2
  - and $\eta_s \in Dist(S)$ is the Dirac (point) distribution on $s \in S$

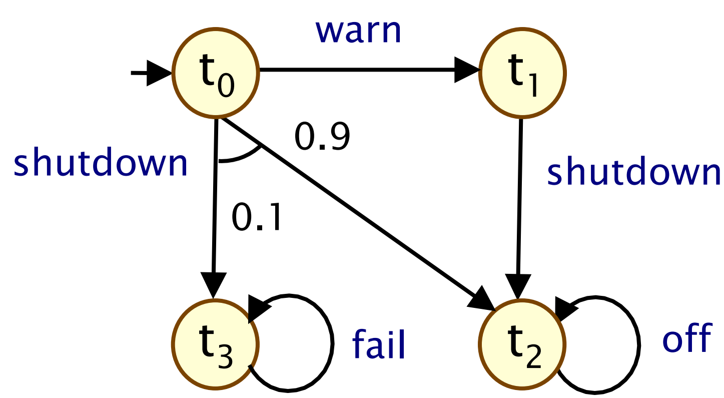
Running example

- Two components, each a Markov decision process:
  - M_1: controller which shuts down devices (after warning first)
  - M_2: device to be shut down (may fail if no warning sent)

MDP M_1 ("controller")

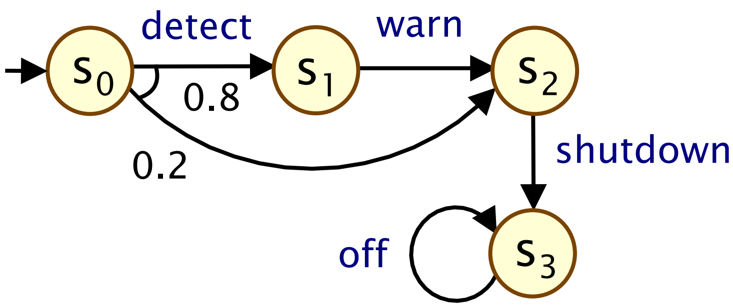


MDP M_2 ("device")

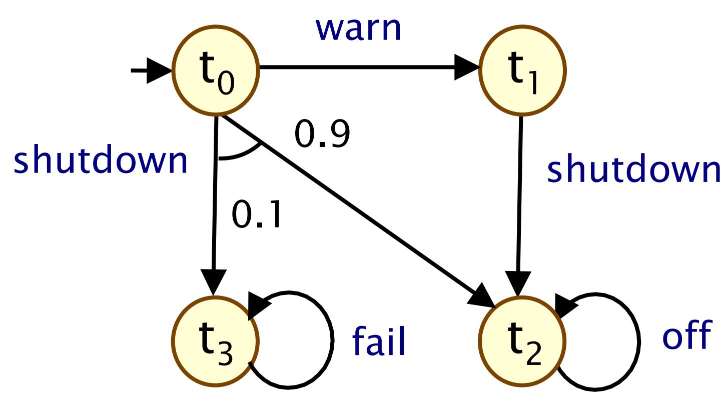


Running example

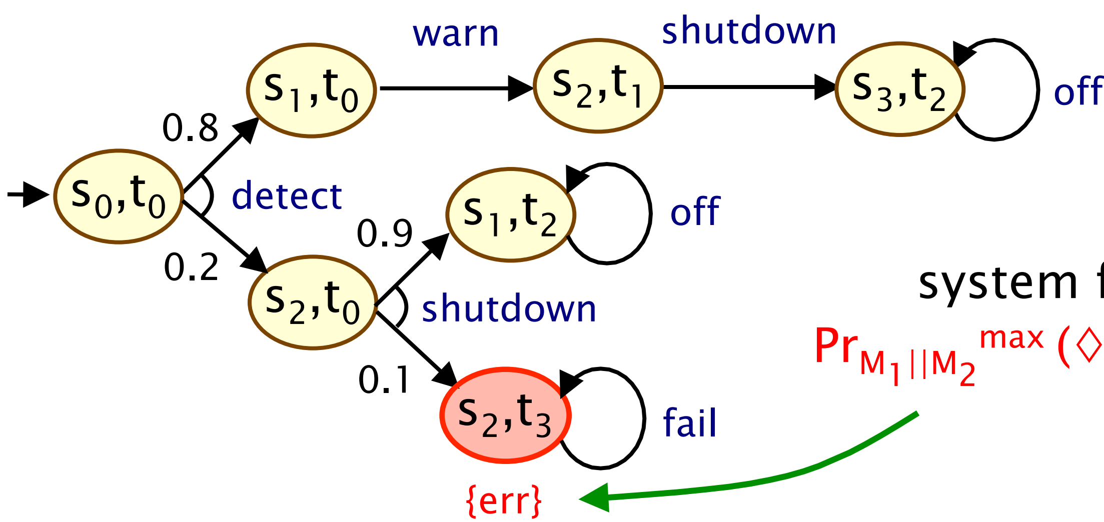
MDP M_1 ("controller")



MDP M_2 ("device")



Parallel composition: $M_1 \,||\, M_2$

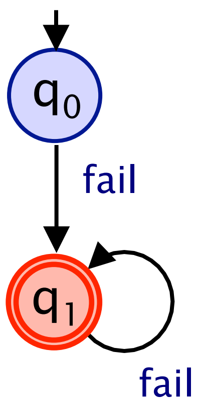


system failure:

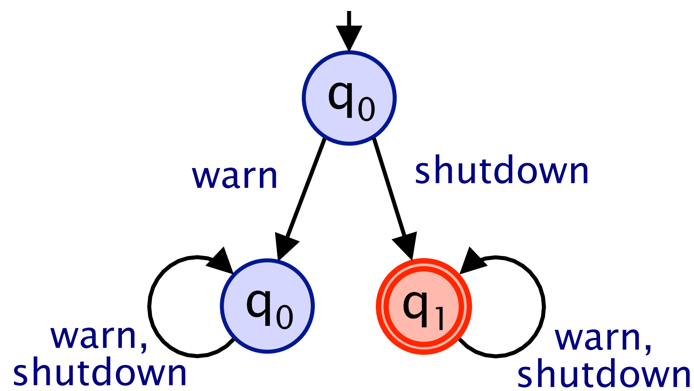
$Pr_{M_1||M_2}^{max}(\Diamond err) = 0.02$

Safety properties

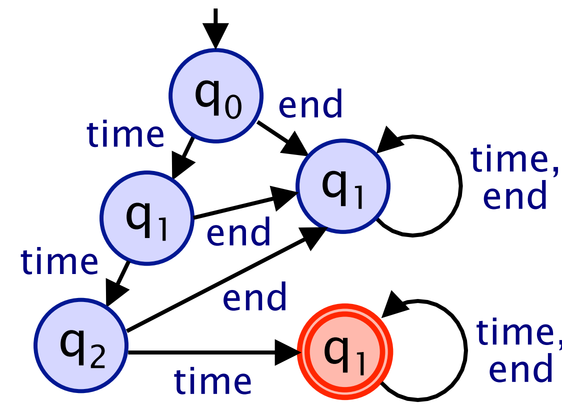
- Safety property: language of infinite words (over actions)
  - characterised by a set of "bad prefixes" (or "finite violations")
  - i.e. finite words of which any extension violates the property
- Regular safety property
  - bad prefixes are represented by a regular language
  - property A stored as deterministic finite automaton (DFA) A_{err}



"a fail action never occurs"



"warn occurs before shutdown"



"at most 2 time steps pass before termination"

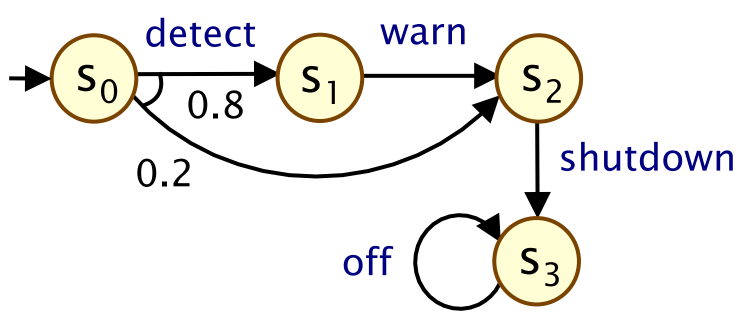
Probabilistic safety properties

- A probabilistic safety property $P_{\geq p}$ [A] comprises
  - a regular safety property A + a rational probability bound p
  - "the probability of satisfying A must be at least p"
  - $M \models P_{\geq p}[A] \;\Leftrightarrow\; Pr_M^{\sigma}(A) \geq p$ for all $\sigma \in Adv_M \;\Leftrightarrow\; Pr_M^{min}(A) \geq p$
- Examples:
  - "*warn* occurs before *shutdown* with probability at least 0.8"
  - "the probability of a failure occurring is at most 0.02"
  - "probability of terminating within k time-steps is at least 0.75"
- Model checking: $Pr_M^{min}(A) = 1 - Pr_{M \otimes A_{err}}^{max}(\Diamond err_A)$
  - where err_A denotes "accept" states for DFA A
  - i.e. construct (synchronous) MDP-DFA product $M \otimes A_{err}$
  - then compute reachability probabilities on product MDP

Running example

- Does probabilistic safety property $P_{\geq 0.8}$ [A] hold in M_1?

MDP M_1 ("controller")



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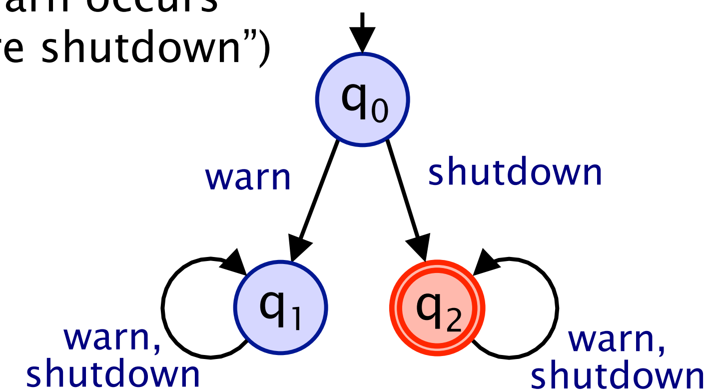

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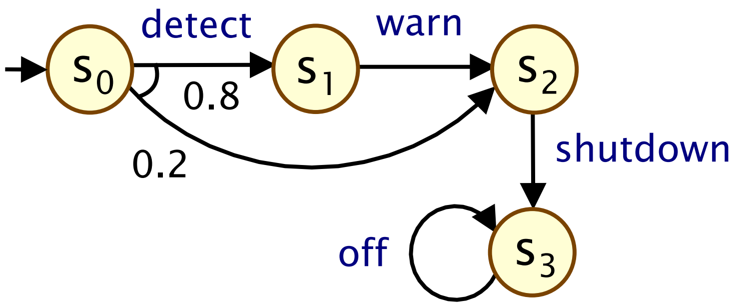

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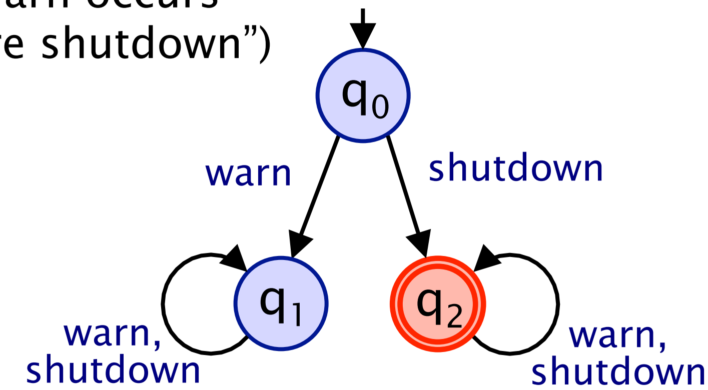
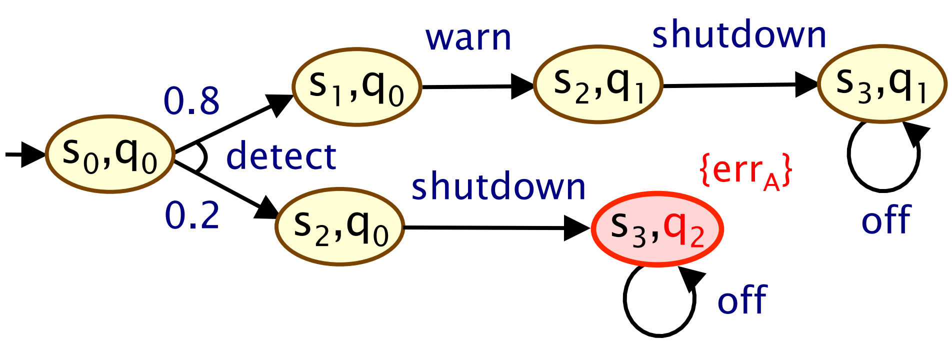


Product MDP $M_1 \otimes A_{err}$



$$Pr_{M_1}^{min}(A)$$
$$= 1 - Pr_{M_1 \otimes A_{err}}^{max}(\Diamond err_A)$$
$$= 1 - 0.2$$
$$= 0.8$$
$$\rightarrow M_1 \models P_{\geq 0.8} [A]$$

Multi-objective MDP model checking

- Consider multiple (linear-time) objectives for an MDP M
  - LTL formulae $\Phi_1,\ldots,\Phi_k$ and probability bounds $\sim_1 p_1,\ldots,\sim_k p_k$
  - question: does there <u>exist</u> an adversary $\sigma \in Adv_M$ such that:

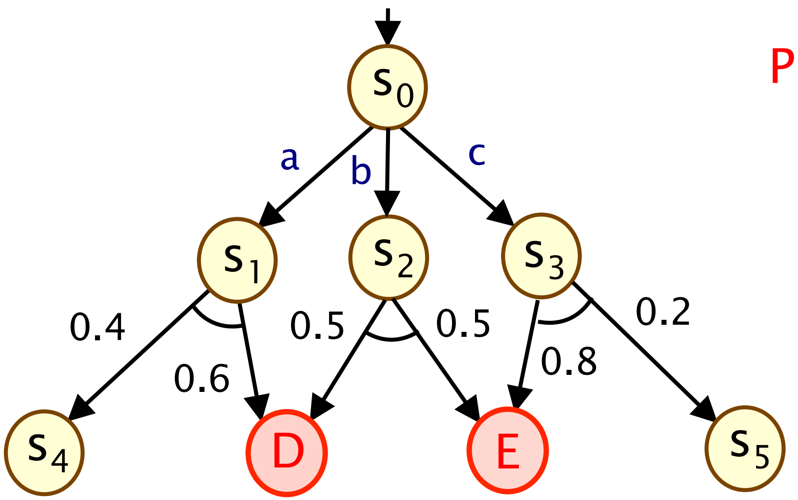
    $Pr_M^\sigma(\phi_1) \sim_1 p_1 \wedge \ldots \wedge Pr_M^\sigma(\phi_k) \sim_k p_k$

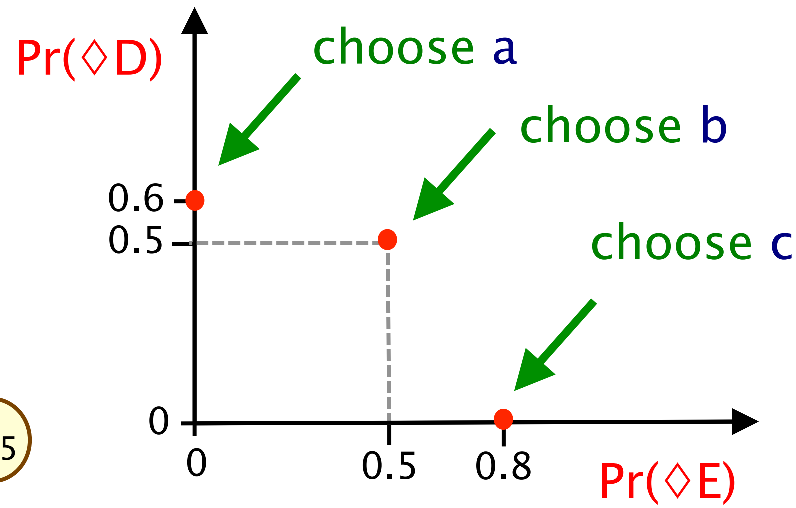
- Motivating example:
  - $Pr_M^\sigma(\Box(queue\_size<10)) > 0.99 \wedge Pr_M^\sigma(\Diamond flat\_battery) < 0.01$

- Multi-objective MDP model checking [EKVY07]
  - construct product of automata for M, $\Phi_1,\ldots,\Phi_k$
  - then solve linear programming (LP) problem
  - the resulting adversary σ can obtained from LP solution
  - note: σ may be randomised (unlike the single objective case)

Multi-objective MDP model checking

- Consider the objectives $\Diamond D$ and $\Diamond E$ in the MDP below
  - i.e. the probability of reaching either state D or E
  - a (randomised) adversary resolves the choice between a/b/c
  - increasing the probability of reaching one target decreases the probability of reaching the other





Multi-objective MDP model checking

- Consider the objectives ◊D and ◊E in the MDP below
  - i.e. the probability of reaching either state D or E
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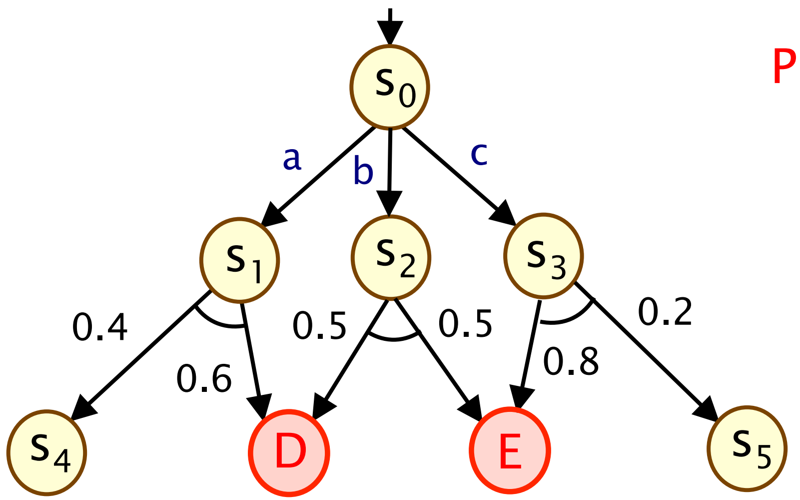

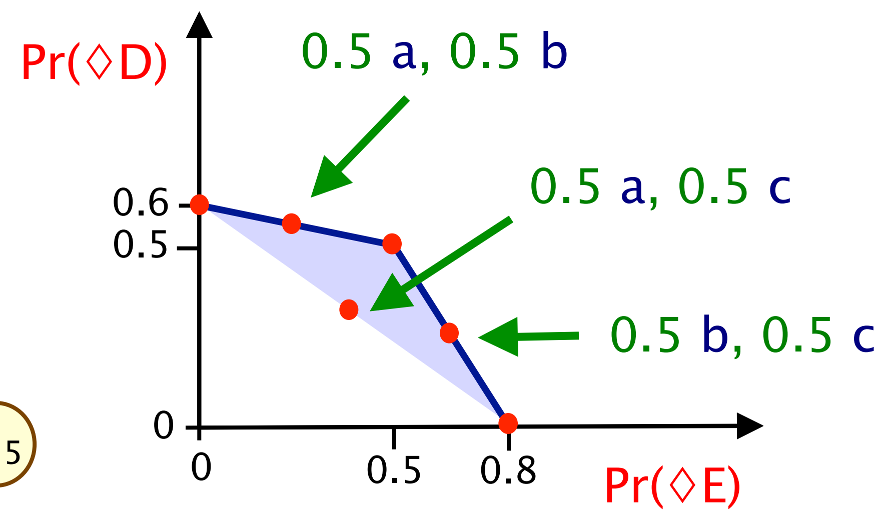

- Considering also randomised adversaries…
  - we obtain a Pareto curve, showing trade-off of optimal solutions

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- Probabilistic assume guarantee
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Probabilistic assume guarantee

- Assume-guarantee triples $\langle A \rangle_{\geq p_A}\ M\ \langle G \rangle_{\geq p_G}$ where:
  - M is a Markov decision process
  - $P_{\geq p_A}[A]$ and $P_{\geq p_G}[G]$ are probabilistic safety properties

- Informally:
  - "whenever M is part of a system satisfying A with probability at least p_A, then the system is guaranteed to satisfy G with probability at least p_G"

- Formally:

$$\langle A \rangle_{\geq p_A}\ M\ \langle G \rangle_{\geq p_G}$$

$$\Leftrightarrow$$

$$\forall \sigma \in Adv_{M[\alpha_A]}\ (\ Pr_{M[\alpha_A]}^{\sigma}(A) \geq p_A \rightarrow Pr_{M[\alpha_A]}^{\sigma}(G) \geq p_G\ )$$

  - where $M[\alpha_A]$ is M with its alphabet extended to include α_A

Assume-guarantee model checking

- Checking whether $\langle A\rangle_{\geq p_A}\ M\ \langle G\rangle_{\geq p_G}$ is true
  - reduces to multi-objective model checking
  - on the product MDP $M' = M[\alpha_A] \otimes A_{err} \otimes G_{err}$
- More precisely:
  - check no adv. of M satisfying $Pr_M^{\sigma}(A) \geq p_A$ but not $Pr_M^{\sigma}(G) \geq p_G$

$$\langle A\rangle_{\geq p_A}\ M\ \langle G\rangle_{\geq p_G}$$
$$\Leftrightarrow$$
$$\neg \exists \sigma' \in Adv_{M'}\ (\ Pr_{M'}^{\sigma'}(\Diamond err_A) \leq 1-p_A \wedge Pr_{M'}^{\sigma'}(\Diamond err_G) > 1-p_G\ )$$

  - solve via LP problem, i.e. in time polynomial in $|M| \cdot |A_{err}| \cdot |G_{err}|$
- Note: $\langle true\rangle\ M\ \langle G\rangle_{\geq p_G}$ denotes the absence of an assumption
  - reduces to standard model checking (since a safety property)

An assume-guarantee rule

- The following asymmetric proof rule holds
  - (symmetric = uses a single assumption about one component)

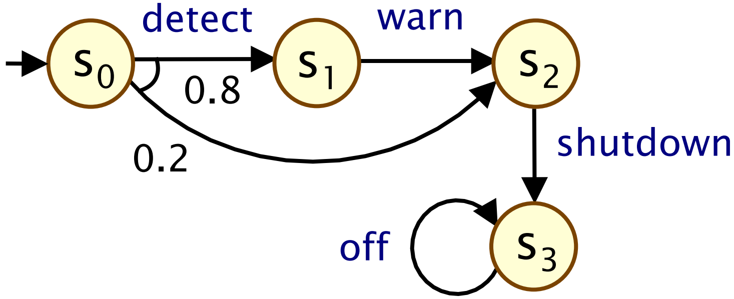
$$\frac{\langle \text{true} \rangle\ M_1\ \langle A \rangle_{\geq p_A} \qquad \langle A \rangle_{\geq p_A}\ M_2\ \langle G \rangle_{\geq p_G}}{\langle \text{true} \rangle\ M_1 \,||\, M_2\ \langle G \rangle_{\geq p_G}} \quad (\text{ASYM})$$

- So, verifying $M_1 \,||\, M_2 \vDash P_{\geq p_G}[G]$ requires:
  - premise 1: $M_1 \vDash P_{\geq p_A}[A]$ (standard model checking)
  - premise 2: $\langle A \rangle_{\geq p_A}\ M_2\ \langle G \rangle_{\geq p_G}$ (multi-objective model checking)
- Potentially much cheaper if $|A|$ much smaller than $|M_1|$

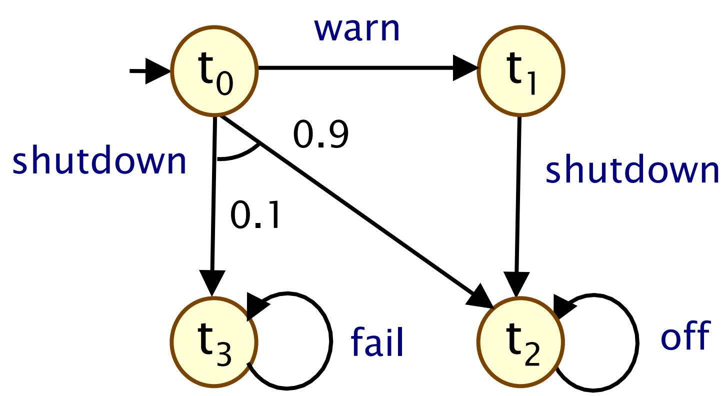
Running example

- Does probabilistic safety property $P_{\geq 0.98}$ [G] hold in $M_1||M_2$?

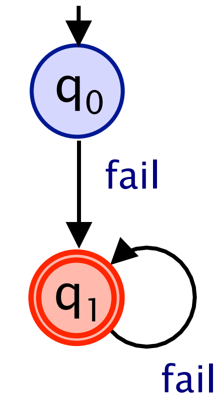
MDP M_1 (“controller”)



MDP M_2 (“device”)



G (“a fail action never occurs”)



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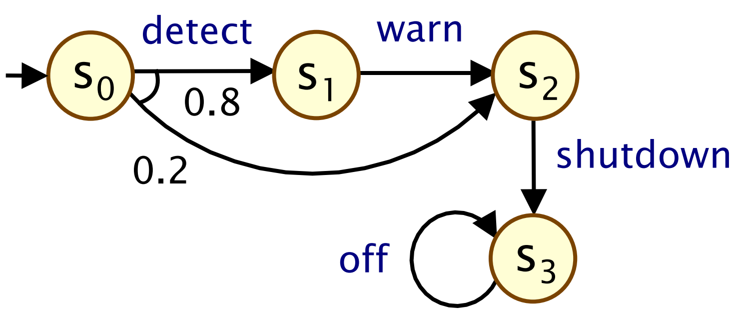
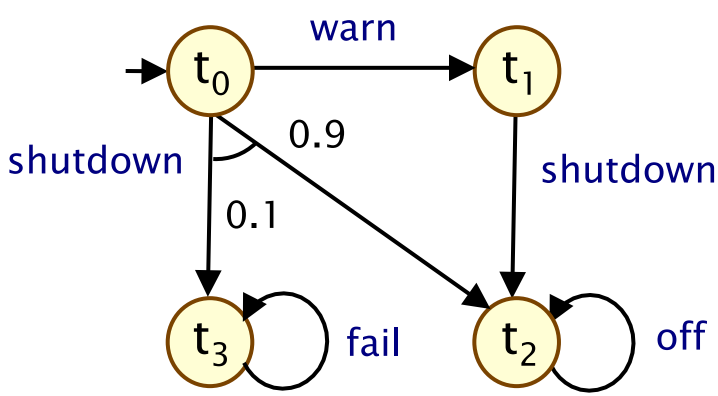


MDP M_2 ("device")



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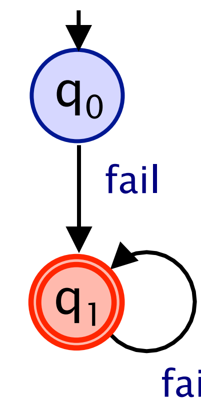
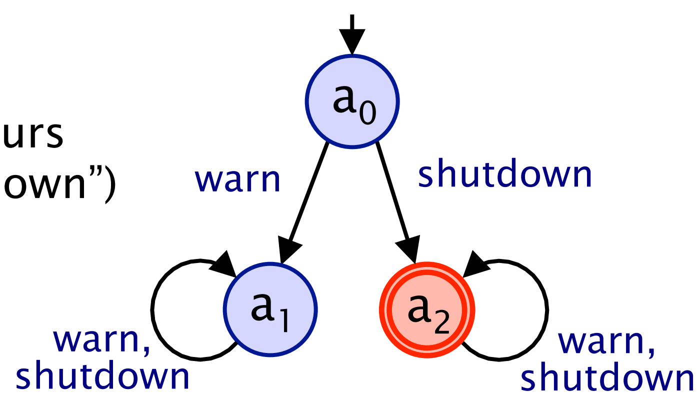


- Use AG with assumption $\langle A\rangle_{\geq 0.8}$ about M_1

$$\frac{\langle \text{true}\rangle\ M_1\ \langle A\rangle_{\geq 0.8} \qquad \langle A\rangle_{\geq 0.8}\ M_2\ \langle G\rangle_{\geq 0.98}}{\langle \text{true}\rangle\ M_1\ ||\ M_2\ \langle G\rangle_{\geq 0.98}}$$

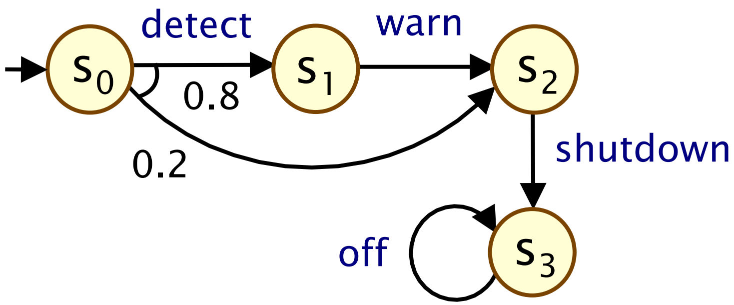
A ("warn occurs before shutdown")



Running example

- Premise 1: Does $M_1 \models P_{\geq 0.8}$ [A] hold? (same as earlier ex.)

MDP M_1 ("controller")



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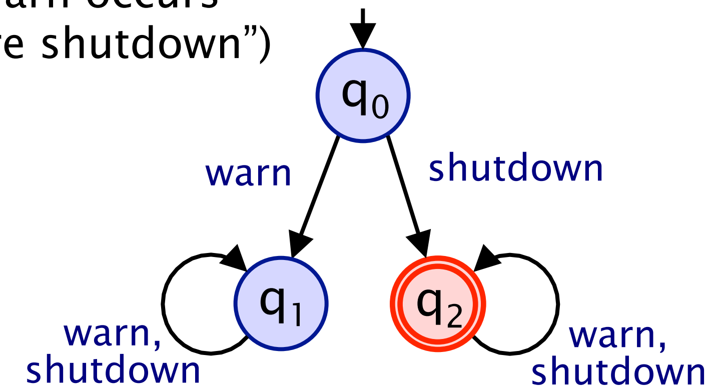
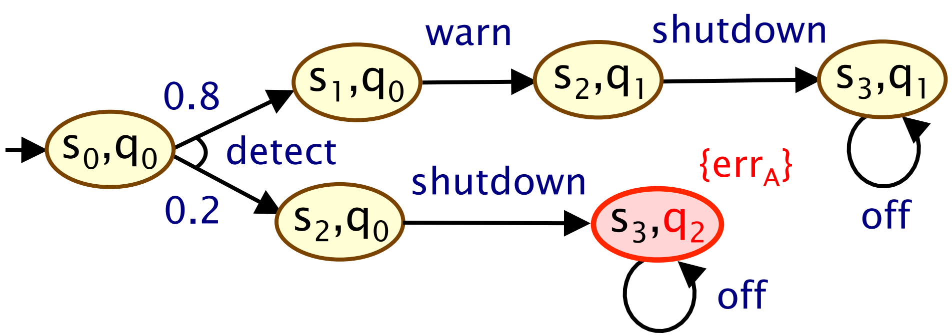


Product MDP $M_1 \otimes A_{err}$

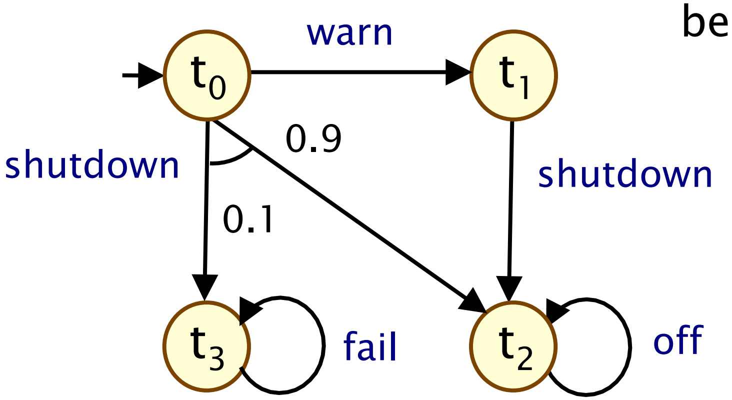


$$Pr_{M_1}{}^{min}(A)$$
$$= 1 - Pr_{M_1 \otimes A_{err}}{}^{max}(\lozenge err_A)$$
$$= 1 - 0.2$$
$$= 0.8$$
$$\rightarrow M_1 \models P_{\geq 0.8} [A]$$

Running example

- Premise 2: Does $\langle A \rangle_{\geq 0.8}\ M_2\ \langle G \rangle_{\geq 0.98}$ hold?

MDP M_2 ("device")



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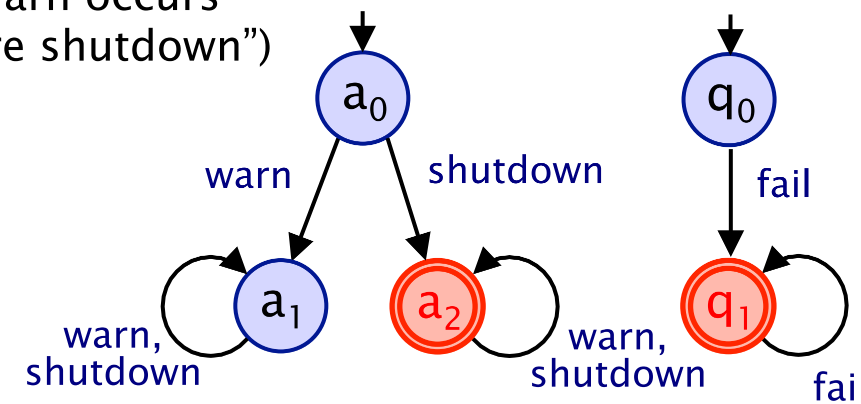
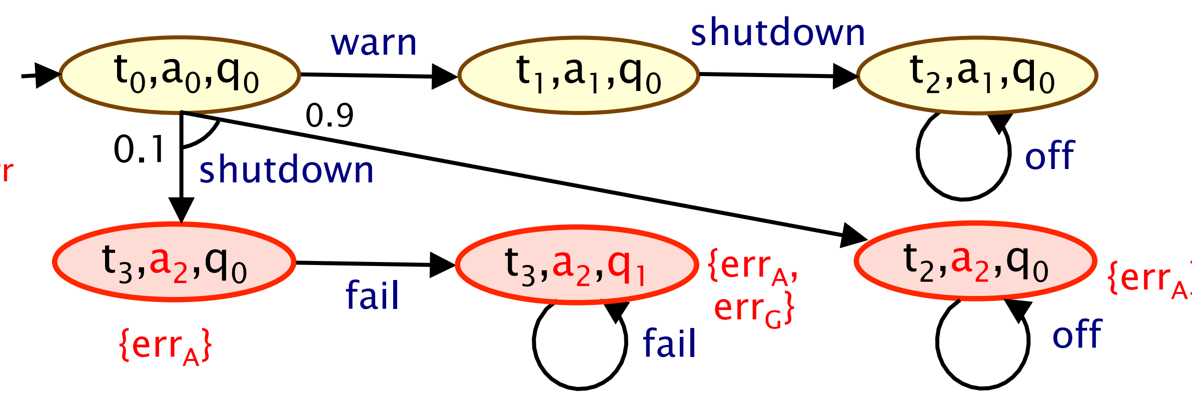


Product MDP

$M' = M_2[\alpha_A] \otimes A_{err} \otimes G_{err}$

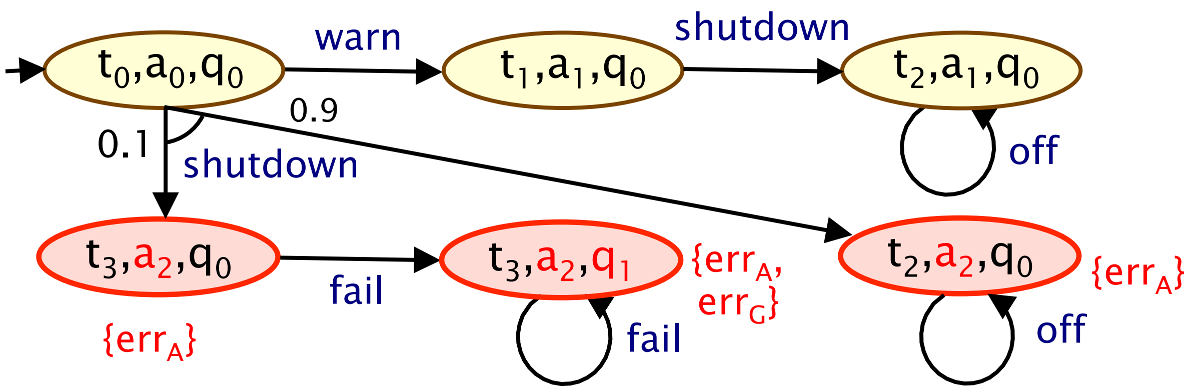


Running example

- Premise 2: Does $\langle A \rangle_{\geq 0.8}\ M_2\ \langle G \rangle_{\geq 0.98}$ hold?

Product MDP

$M' = M_2[\alpha_A] \otimes A_{err} \otimes G_{err}$



- $\exists$ an adversary of M_2 satisfying $Pr_M^{\sigma}(A) \geq 0.8$ but not $Pr_M^{\sigma}(G) \geq 0.98$?

  $\Leftrightarrow$

- $\exists$ an an adversary of M' with $Pr_{M'}^{\sigma'}(\Diamond err_A) \leq 0.2$ and $Pr_{M'}^{\sigma'}(\Diamond err_G) > 0.02$?

- To satisfy $Pr_{M'}^{\sigma'}(\Diamond err_A) \leq 0.2$, adversary σ' must choose shutdown in initial state with probability ≤ 0.2, which means $Pr_{M'}^{\sigma'}(\Diamond err_G) \leq 0.02$

- So, there is no such adversary and $\langle A \rangle_{\geq 0.8}\ M_2\ \langle G \rangle_{\geq 0.98}$ <u>does</u> hold

Other assume-guarantee rules

Multiple assumptions:

$$\frac{\langle true \rangle\ M_1\ \langle A_1,\ldots,A_k \rangle_{\geq p_1,\ldots,p_k} \qquad \langle A_1,\ldots,A_k \rangle_{\geq p_1,\ldots,p_k}\ M_2\ \langle G \rangle_{\geq p_G}}{\langle true \rangle\ M_1\ ||\ M_2\ \langle G \rangle_{\geq p_G}}$$

Multiple components (chain):

$$\frac{\langle true \rangle\ M_1\ \langle A_1 \rangle_{\geq p_1} \qquad \langle A_1 \rangle_{\geq p_1}\ M_2\ \langle A_2 \rangle_{\geq p_2} \qquad \ldots \qquad \langle A_n \rangle_{\geq p_n}\ M_n\ \langle G \rangle_{\geq p_G}}{\langle true \rangle\ M_1\ ||\ \ldots\ ||\ M_n\ \langle G \rangle_{\geq p_G}}$$

Circular rule:

$$\frac{\langle true \rangle\ M_2\ \langle A_1 \rangle_{\geq p_2} \qquad \langle A_2 \rangle_{\geq p_2}\ M_1\ \langle A_1 \rangle_{\geq p_1} \qquad \langle A_1 \rangle_{\geq p_1}\ M_2\ \langle G \rangle_{\geq p_G}}{\langle true \rangle\ M_1\ ||\ M_2\ \langle G \rangle_{\geq p_G}}$$

Asynchronous components:

$$\frac{\langle A_1 \rangle_{\geq p_1}\ M_1\ \langle G_1 \rangle_{\geq q_1} \qquad \langle A_2 \rangle_{\geq p_2}\ M_2\ \langle G_2 \rangle_{\geq q_2}}{\langle A_1, A_2 \rangle_{\geq p_1 p_2}\ M_1\ ||\ M_2\ \langle G_1 \vee G_2 \rangle_{\geq (q_1 + q_2 - q_1 q_2)}}$$

A quantitative approach

- For (non-compositional) probabilistic verification
  - prefer quantitative properties: $Pr_M^{min}(G)$, not $M \models P_{\geq p_G} [G]$
  - can we do this for compositional verification?

$$\frac{\langle true \rangle\ M_1\ \langle A \rangle_{\geq p_A} \qquad \langle A \rangle_{\geq p_A}\ M_2\ \langle G \rangle_{\geq p_G}}{\langle true \rangle\ M_1 \,||\, M_2\ \langle G \rangle_{\geq p_G}}$$

- Consider, for example, AG rule (ASYM)
  - this proves $Pr_{M_1 \| M_2}^{min}(G) \geq p_G$ for certain values of p_G
  - i.e. gives lower bound for $Pr_{M_1 \| M_2}^{min}(G)$
  - for a fixed assumption A, we can compute the maximal lower bound obtainable, through a simple adaption of the multi-objective model checking problem
  - we can also compute upper bounds using generated adversaries as witnesses
  - furthermore: can explore trade-offs in parameterised models by approximating Pareto curves

Implementation + Case studies

- Prototype extension of PRISM model checker
  - already supports LTL for Markov decision processes
  - automata can be encoded in modelling language
  - added support for multi-objective LTL model checking, using LP solvers (ECLiPSe/COIN-OR CBC)

- Two large case studies
  - randomised consensus algorithm (Aspnes & Herlihy)
    - minimum probability consensus reached by round R
  - Zeroconf network protocol
    - maximum probability network configures incorrectly
    - minimum probability network configured by time T

Case study: Randomised consensus

- Distributed consensus protocol
  - algorithm run by a collection of distributed processes
  - processes each have some (nondeterministic) initial value
  - processes must eventually terminate, agreeing on same value

- Aspnes/Herlihy randomised distributed consensus [AH90]
  - consensus algorithm for N processes, operates in rounds
  - each round uses a shared coin protocol, parameterised by K

- We check:
  - "minimum probability consensus reached by round R"
  - captured as a probabilistic safety property with DFA representing any run where a process enters round R+1

Case study: Randomised consensus

- Model structure: parallel composition of:
  - N MDPs, each representing one process
  - R MDPs, one for the shared coin protocol of each round

- Compositional verification:
  - model check a probabilistic safety property for each coin protocol from rounds 1, . . . , $R-2$
  - safety property: minimum probability that the coin protocol returns the same coin value for all processes
  - combine these results through $R-2$ applications of of the “asynchronous” rule, proving a probabilistic safety property about the parallel composition of the R-2 coin protocols
  - this probabilistic safety property is used as the assumption for an application of the (ASYM) rule, yielding the final property

Experimental results

Case study [parameters]		Non-compositional		Compositional	
		States	Time (s)	LP size	Time (s)
Randomised consensus (3 processes) [R,K]	3, 2	1,418,545	18,971	40,542	29.6
	3, 20	39,827,233	time-out	40,542	125.3
	4, 2	150,487,585	78,955	141,168	376.1
	4, 20	2,028,200,209	mem-out	141,168	471.9
ZeroConf [K]	4	313,541	103.9	20,927	21.9
	6	811,290	275.2	40,258	54.8
	8	1,892,952	592.2	66,436	107.6
ZeroConf time-bounded [K, T]	2, 10	65,567	46.3	62,188	89.0
	2, 14	106,177	63.1	101,313	170.8
	4, 10	976,247	88.2	74,484	170.8
	4, 14	2,288,771	128.3	166,203	430.6

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- Faster than conventional model checking in a number of cases

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- Verified instances where conventional model checking is infeasible

Experimental results

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	4, 14	2,288,771	128.3	166,203	430.6

- LP problem generally much smaller than full state space (but still the limiting factor)

Overview (Part 3)

- Compositional verification
  - assume-guarantee reasoning
- Markov decision processes
  - probabilistic safety properties
  - multi-objective model checking
- Probabilistic assume guarantee
  - semantics, model checking
  - assume-guarantee proof rules
  - quantitative approaches
  - implementation & experimental results
  - assumption generation with learning

Generating assumptions

- We can verify $M_1 || M_2$ compositionally
  - but this relies on the existence of a suitable assumption $\langle A \rangle_{\geq p_A}$

$$\frac{\langle true \rangle\ M_1\ \langle A \rangle_{\geq p_A} \qquad \langle A \rangle_{\geq p_A}\ M_2\ \langle G \rangle_{\geq p_G}}{\langle true \rangle\ M_1\ ||\ M_2\ \langle G \rangle_{\geq p_G}}$$

- 1. Does such an assumption always exist?
- 2. When it does exist, can we generate it automatically?

- One possibility: use algorithmic learning techniques
  - inspired by non-probabilistic AG work of [Pasareanu et al.]
  - uses L* algorithm to learn finite automata for assumptions
  - successful implementations using Boolean functions [Chen/Clarke/et al.] and BDD-based techniques [Alur et al.]

- We use a modified version of L*
  - to learn probabilistic assumptions for rule (ASYM)

L* for assume-guarantee

- L* algorithm [Angluin] – learns regular languages (as a DFA)
  - relies on existence of a "teacher" to guide the learning
  - answers two type of queries: "membership" and "conjecture"
  - membership: "is word w in the target language L?"
  - conjecture: "does automata A accept the target language L"?
  - if not, teacher must return counterexample w'
  - L* produces minimal DFA, runs in polynomial time

- Successfully applied to the of learning assumptions for AG
  - uses notion of "weakest assumption" about a component that suffices for compositional verification (always exists)
  - weakest assumption is the target regular language
  - model checker plays role of teacher, returns counterexamples
  - in practice, can usually stop early: either with a simpler (stronger) assumption or by refuting the property

Key steps of (modified) L*

- Key idea: learn probabilistic assumption $\langle A \rangle_{\geq p_A}$
  - via non-probabilistic assumption A

$$\frac{\langle true \rangle\ M_1\ \langle A \rangle_{\geq p_A} \quad \langle A \rangle_{\geq p_A}\ M_2\ \langle G \rangle_{\geq p_G}}{\langle true \rangle\ M_1 \mid\mid M_2\ \langle G \rangle_{\geq p_G}}$$

- "Membership" query (for trace t):
  - does $t \mid\mid M_2 \models P_{\geq p_G}[G]$ hold?

- "Conjecture" query (for assumption A)
  - 1. compute lowest value of p_A such that $\langle A \rangle_{\geq p_A}\ M_2\ \langle G \rangle_{\geq p_G}$ holds
    - if no such value, need to refine A
  - 2. check if $M_1 \models P_{\geq p_A}[A]$ holds
    - if yes, successfully verified $\langle G \rangle_{\geq p_G}$ for $M_1 \mid\mid M_2$ (with $\langle A \rangle_{\geq p_A}$)
  - 3. check if counterexample from 2 is real
    - if yes, have refuted $\langle G \rangle_{\geq p_G}$ for $M_1 \mid\mid M_2$
    - if no, need to refine A
  - (use probabilistic counterexamples [HK07] to "refine A")

Experimental results (learning)

Case study [parameters]		Component sizes		Compositional	
		$\|M_2 \otimes G_{err}\|$	$\|M_1\|$	$\|A\|$	Time (s)
Client-server (N failures) [N]	3	229	16	4	6.6
	4	1,121	25	5	13.1
	5	5,397	36	6	87.5
Randomised consensus [N,R,K]	2, 3, 20	391	3,217	5	24.2
	2, 4, 2	573	113,569	10	108.4
	3, 3, 2	8,843	4,065	14	681.7
	3, 3, 20	8,843	38,193	14	863.8
Sensor network [N]	1	42	72	2	3.5
	2	42	1,184	2	3.7
	3	42	10,662	2	4.6

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- Successfully learnt (small) assumptions in all cases

Experimental results (learning)

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- In some cases, learning + compositional verification is faster (than non-compositional verification, using PRISM)

Summary (Part 3)

- Compositional verification, e.g. assume-guarantee
  - decompose verification problem based on system structure
- Compositional probabilistic verification based on:
  - Markov decision processes, with arbitrary parallel composition
  - assumptions/guarantees are probabilistic safety properties
  - reduction to multi-objective model checking
  - multiple proof rules; adapted to quantitative approach
  - automatic generation of assumptions: L* learning
- Can work well in practice
  - verified safety/performance on several large case studies
  - cases where infeasible using non-compositional verification
- For further detail, see [KNPQ10], [FKP10]

- Next: Probabilistic timed automata (PTAs)